Abstract—Numerical study has been conducted to understand the gas flows in a conical nozzle at different degree of angle using 2 dimensional axi-symmetric models, which solves the governing equations by a control volume method. The nozzle geometry co-ordinates are taken by using of method of characteristics which usually designed for De-Laval nozzle. The present study is aimed at investigating the supersonic flow in conical nozzle for Mach 3 at various degree of angle. The throat diameter and exit diameter is same for all nozzles. The flow is simulated using Fluent software. The flow parameters, like pressure ratio, Area of nozzle exit ratio, and the Mach number of the flow at the nozzle exit is defined prior to the simulation. The result shows the variation in the Mach no., pressure ratio. For 40 angles the Mach number at exit is very low compared to other nozzles and the turbulence intensity is very high for 160 at exit and it is nearly 3.045e+05. While when the angle is 80 the mach number at nozzle exit is 2.91 and same as 120 angle but an angle 160 it gives the mach number at nozzle exit is 2.92 and it is lowest at an angle 40. The degree of angle for conical nozzle can be large as 12 to 18 degree maximum so for maximum thrust we can go with 120 or 160 conical nozzle.

Index Terms—Conical nozzle, degree of angle, supersonic flow, Mach number, Pressure ratio, Control Volume

I. INTRODUCTION

Swedish engineer of French descent who, in trying to develop a more efficient steam engine, designed a turbine that was turned by jets of steam. The critical component – the one in which heat energy of the hot high-pressure steam from the boiler was converted into kinetic energy – was the nozzle from which the jet blew onto the wheel. De Laval found that the most efficient conversion occurred when the nozzle first narrowed, increasing the speed of the jet to the speed of sound, and then expanded again. Above the speed of sound (but not below it) this expansion caused a further increase in the speed of the jet and led to a very efficient conversion of heat energy to motion. The theory of air resistance was first proposed by Sir Isaac Newton in 1726. According to him, an aerodynamic force depends on the density and velocity of the fluid, and the shape and the size of the displacing object. Newton’s theory was soon followed by other theoretical solution of fluid motion problems. All these were restricted to flow under idealized conditions, i.e. air was assumed to possess constant density and to move in response to pressure and inertia.

Nowadays steam turbines are the preferred power source of electric power stations and large ships, although they usually have a different design–to make best use of the fast steam jet, de Laval’s turbine had to run at an impractically high speed. But for rockets the de Laval nozzle was just what was needed.

II. MATERIAL AND METHODS  MATHEMATICAL MODEL

A mathematical model comprises equations relating the dependent and the independent variables and the relevant parameters that describe some physical phenomenon. Typically, a mathematical model consists of differential equations that govern the behavior of the physical system, and the associated boundary conditions.

To start with fluent, it is necessary to know if the meshed geometry is correct, so is checked. To ensue with, we are to define the model, material, operating condition and boundary condition. Models are to be set in order to define if any energy equation is dealt with our study, if the flow is viscous etc. We have chosen coupled solver, 2d implicit, absolute velocity formulation, cell based gradient option, superficial velocity porous formulation. As our flow is dealt with energy equation so is necessary to check them up. The material is selected as air and the density as ideal gas to make the solution simpler. Under the solve command the control is selected for limiting the pressure to a maximum of 5e+7 and minimum of 1e+4. The initialization of value is computed from the inlet. It is also necessary to select the appropriate approximation required in the residual command under monitors and check in plot to visualize the progress of iteration. Once every parameter is described the iteration is performed till the value gets converged to required approximation. The figures can be plotted between position in x-axis and any other function in y-axis from plot command or else to view vectors, contours or grid display command is to be chosen.

III. RESULTS AND DISCUSSION

The geometry of the grid is designed by the help of gambit tool and the figure 4.1 is made by using of method of
characteristics which usually designed for De-Laval nozzle. On the same basic we take the throat co-ordinates and exit co-ordinates for designing this nozzle and after calculating, we get an angle nearly $7.128567^\circ$ and simulate it using Fluent software and after this we take minimum degree of angle for conical nozzle i.e. $4^\circ$ (Fig. 4.2) and check the variation in Mach Number and other properties too.

Fig 4.1 Mach Number 3 Default Angle = $7.128576^\circ$

From the above figure, it is clearly visualized that in the convergent section at inlet point, Mach number, is in the Sub-sonic region ($=0.399$) while at the throat, flow becomes Sonic ($=1.02$) and at the nozzle exit it becomes Super-Sonic ($=2.90$) for which the nozzle is designed. Near the wall, the Mach number is 1.65. This is due to the viscosity and turbulence in the fluid. STATIC PRESSURE: -

Fig 4.2 Mach Number 3 Angle = 40

Fig 4.3 Mach Number 3 Angle = 80

Fig 4.4 Mach Number 3 Angle = 120

Grid Arrangement Mach 3 16$^\circ$ angle: -

Grid Arrangement Mach 3 16$^\circ$ angle: -

Fig 4.5 Mach Number 3 Angle = 160

MACH 3 DEFAULTS: - ANGLE = $7.128576^\circ$

MACH NUMBER: -

Fig. 5.1 Mach number Angle = $7.1285760$

In this, the nozzle is designed for Mach no. 3. From figure, it is clearly visualized that in the convergent section at inlet point, Mach number, is in the Sub-sonic region ($=0.399$) while at the throat, flow becomes Sonic ($=1.02$) and at the nozzle exit it becomes Super-Sonic ($=2.90$) for which the nozzle is designed. Near the wall, the Mach number is 1.65. This is due to the viscosity and turbulence in the fluid. STATIC PRESSURE: -

Fig. 5.2 Static Pressure Angle = $7.1285760$

Static pressure is the pressure that is exerted by a fluid. Specifically, it is the pressure measured when the fluid is still, or at rest. The above figure reveals the fact that the gas gets expanded in the nozzle exit. The static pressure in the inlet is observed to be $3.23e+05$ Pa and as we move towards the throat there is a decrease and the value at the throat is found out to be $2.30e+05$ Pa. After the throat, there is sudden expansion and the static pressure falls in a more rapid manner towards the exit of the nozzle. At the exit it is found to be $1.13e+04$ Pa. TOTAL PRESSURE: -

Fig. 5.3 Total Pressure Angle = $7.1285760$

From the above figure, you can easily visualize in the above figure that, there is decrease in stagnation pressure near the nozzle walls due to viscous effects, whereas the stagnation pressure remains almost constant in the centre. The value throughout is found to be $3.63e+05$ Pa except near the walls where it decreases to $5.88e+04$ Pa. TOTAL TEMPERATURE: -

Fig. 5.4 Total Temperature Angle = $7.1285760$

The total temperature almost remains a constant in the inlet up to the throat after which it tends to increase. Near the walls the temperature decreases to $5.92e+02$ K. In the inlet and the throat the temperature is $6.00e+02$ K. After the throat, the temperature increases to $6.03e+02$ K at the exit. As we move from the centre vertically upwards at the exit, there is variation. At the centre it is $6.02e+02$ K, at the walls it is $5.92e+02$ K and moving inward a little bit from the wall the temperature reaches a maximum of $6.03e+02$ K. VELOCITY MAGNITUDE: -

Fig. 5.5 Velocity Magnitude Angle = $7.1285760$

There is symmetric flow as observed from the above figure. There is a constant increase in the velocity magnitude of the fluid as we move from the inlet of the convergent section ($=1.74e+02$ m/s) to the throat ($=3.92e+02$ m/s) and to the exit of the divergent section ($=8.70e+02$ m/s). The flow is turbulent, so near the wall flow separation takes place.
causing the velocity to decrease to nearly 8.70e+01 m/s. The maximum velocity is reached at the exit.

**TURBULENT INTENSITY:**

The turbulent intensity at the convergent section is very low (=1.04e+03 %). Almost till the throat it remains almost a constant. At the throat there is a very small increase to 1.25e+03 %. As soon as it crosses the throat, there is a sudden increase in the turbulent intensity due to the sudden increase in the area. As we move towards the exit, there is a small patch in the centre where the turbulent intensity increases (2.08e+03 %) and then as the fluid stabilizes near the exit, there is a decrease in the turbulent intensity (1.87e+03 %). Near the walls the turbulent intensity is high due to the reversals of flow. At the exit near the walls the turbulent intensity reaches a maximum (=4.97e+03 %).

**MACH 3:**

From the figure, it is clearly observed that mach number in the convergent section is 0.399 (Sub-Sonic), at the throat it is 1.02(Sonic) and as we move in the divergent section, it keeps increasing and at the exit it is 2.88(Super-Sonic). Parallel flow is observed which is a characteristic of the conical nozzle and its design purpose (for Mach 3) is also solved. Mach Number near the wall is less due to the viscosity and the turbulence (=1.76). The Mach Number for the default angle turns out to be 2.90 but for and angle of 40 it comes out to be 2.88. This is due to the change in the geometry due to which flow also changes.

**STATIC PRESSURE:**

The above figure shows that the gas gets over expanded in the nozzle exit. The static pressure in the inlet is observed to be 3.24e+05 Pa and the value at the throat is found out to be 2.30e+05 Pa. At the exit the pressure is found to be 1.18e+04 Pa. Right from the inlet to the throat to the exit the static pressure tends to decrease. There is a considerable decrease observed after the throat to the exit where there is a large drop in the static pressure. As compared to the previous case (Fig 5.32) there is a change in the exit static pressure value.

**TOTAL PRESSURE:**

From the above figure, you can easily visualize in the above figure that, there is decrease in stagnation pressure near the nozzle walls. This is due to the viscous effects, whereas the stagnation pressure remains almost constant in the centre. The value uniformly in the centre is found to be 3.64e+05 Pa except near the walls where it decreases to 5.41e+04 Pa. There is a slight increase in the total pressure in the centre as well as the walls when compared to the default angle version of the nozzle.

**TOTAL TEMPERATURE:**

The total temperature almost remains a constant in the inlet up to the throat. Increase is observed after some distance from the throat towards the exit as seen in the above figure. Near the walls the temperature decreases to 5.93e+02 K. In the inlet and the throat the temperature is 6.00e+02 K. After the throat, the temperature increases to 6.04e+02 K at the exit. At the exit, moving vertically upward there is variation. The maximum value is not attained at the centre but at some distance from the centre. At the centre it is 6.01e+02 K and at the wall it is 5.93e+02 K. The maximum value of 6.04e+02 K is attained near the walls but some distance away from it. There is not much difference in the total temperature contour when compared to the default nozzle but the fact that in the default nozzle, the temperature increases as soon as the throat is crossed but in this it is clearly observed that there is a beginning in the rise in temperature after some distance from the throat.

**VELOCITY MAGNITUDE:**
The velocity magnitude as observed from the above figure is that it keeps increasing as we move from the inlet to the exit. The value at inlet is $2.17 \times 10^2$ m/s and then increases to the throat where its value is $3.91 \times 10^2$ m/s. The velocity magnitude then increases to $8.69 \times 10^2$ m/s at the exit. There is a drop over the walls and the value near the walls at the exit is $1.30 \times 10^2$ m/s due to the turbulence which causes reversals in the flow. There is almost no change when compared to that of default mach angle nozzle. The pattern is similar and the value differs by $0.01 \times 10^2$ m/s.

**TURBULENT INTENSITY:**

The turbulent intensity at the convergent section is $1.03 \times 10^3$ % which is low. It remains a constant almost up to the throat. At the throat it increases to $1.23 \times 10^3$ %. Since there is an increase in the area, there is also an increase in the turbulent intensity as we move towards the exit from the throat. There is a difference when compared to the default nozzle. There is a larger increase in the area just after in the default case where as the increase in the area is more gradual in the $4^\circ$ angle case. As we move towards the exit, there is a small patch in the centre extending from the throat to almost the centre between the throat and the exit where the turbulent intensity increases ($2.01 \times 10^3$ %) and as we exit there is stabilization of the fluid near the exit of the nozzle ($1.82 \times 10^3$ %). The turbulent intensity reaches a maximum of $4.97 \times 10^3$ % near the walls as there is reversal of flow and turbulence due to the walls.

**MACH 3: ANGLE = 8^\circ**

From figure it can be seen that there is parallel flow. The Mach Number at the inlet is 0.525 which is Sub-Sonic, 1.03 at the throat and 2.91 at the exit. There is a decrease in the value of Mach Number near the walls. Its value is 1.65. Turbulence and viscosity are the cause for this decrease. The Mach Number has increased in this case to 2.91 whereas it was 2.88 in the case of $4^\circ$.

**STATIC PRESSURE:**

There is a continuous decrease in the value of static pressure from the inlet to the throat and to the exit. At the inlet it is $3.24 \times 10^5$ Pa. It then keeps on decreasing to $2.30 \times 10^5$ Pa which is also observed in the case of $4^\circ$ nozzle. The main difference arises in the exit where there is a deviation in the minimum value. The minimum value in $4^\circ$ nozzle was $1.18 \times 10^4$ Pa whereas here it is observed to be $1.13 \times 10^4$ Pa. Other than that there is no major difference between the two. The pattern is also similar.

**TOTAL PRESSURE:**

The total pressure contour is shown above. The total pressure is similar to that of the $4^\circ$ version. It is uniform throughout the centre and around the walls it is less. It is $3.62 \times 10^5$ Pa in the centre throughout. At the exit, the value differs if we go upwards or downwards vertically. There is a decrease near the walls to $5.99 \times 10^4$ Pa. The maximum value of the stagnation pressure has decreased from $3.64 \times 10^5$ Pa to $3.62 \times 10^5$ Pa from the $4^\circ$ nozzle to this one.

**TOTAL TEMPERATURE:**

The total temperature contour is also similar to the previous version. There is uniformity observed from the inlet to some distance from the throat where it remains a constant whose value is $6.00 \times 10^2$ K. After which it increases to $6.02 \times 10^2$ K in the centre. The maximum value of $6.04 \times 10^2$ K is attained some distance from the wall. There is decrease near the wall. Vertically moving from the centre, it is $6.02 \times 10^2$ K up to more than the upper (or) lower half after which it reaches the maximum value of $6.04 \times 10^2$ K and then it decreases to the wall where it has a value of $5.95 \times 10^2$ K. The variation is similar in the case of $4^\circ$ nozzle also.

**VELOCITY MAGNITUDE:**

There is considerable increase in the velocity profile as observed above. The walls show a lower value because of the turbulence and the reversals in the flow. At inlet the velocity is $2.18 \times 10^2$ m/s and there is increase till the throat up to
3.92e+02 m/s. Then it increases to the exit up to 8.71e+02 m/s. There is a similar trend observed in angle 4° nozzles too. There is just an increase in the maximum value from 8.69e+02 m/s to 8.71e+02 m/s.

**TURBULENT INTENSITY:**

![Graph showing turbulent intensity](image)

The turbulent intensity at the convergent section is 3.48e+02 % which is very low. It remains a constant almost up to the throat. At the throat it increases to 5.92e+02 %. There is an increase in the value after the throat as there is a sudden expansion to a bigger area than the 4° nozzle. There is also a difference in the pattern when compared to the 4° nozzle. There were patches in the 1° 2 nozzles (default and 4°) but here there are no nozzles and the flow stabilization which was observed towards the end of the nozzle in the other nozzles is not observed here as the increase is right from the throat to the exit. The maximum value however is reached near the walls as the reversals cause more turbulence. The exit turbulent Intensity value at the centre is 1.57e+03%.

Moving vertically upward (or) downward at the exit, at the centre it is 1.57e+03 % which remains a constant up to some distance after which there is a slight decrease to 1.08e+03% and again it decrease after which there is a steep increase up to the wall where it attains a maximum value to 4.99e+03 %. There is an increase in the maximum value when compared to the previous nozzle.

**MACH 3: - ANGLE = 12°**

![Graph showing Mach number](image)

The region in the inlet is sub-sonic and has an inlet value of 0.404. As it nears the throat it becomes sonic and at the throat it is 1.03 and the exit is super-sonic and it exits with a Mach Number of 2.91. The motive of the design of the conical nozzle is also achieved with parallel flow. There are irregularities near the walls due to the reversals of flow. The Mach number as observed in the previous case of angle of 8° was also 2.91 which is also the same in the present case.

**STATIC PRESSURE:**

![Graph showing static pressure](image)

The static pressure contour is also similar to that of the previous version. It decreases to the throat and then continues to decrease till the exit. The value at the inlet is 3.23e+05 Pa and at the throat it is 1.67e+05 Pa. There is also a steep decrease after the throat where the static pressure reduces to 1.11e+04 Pa at halfway around outlet and remains the same till the exit. There is a slight decrease in the maximum value of static pressure (0.01e+05 Pa).

**TOTAL PRESSURE:**

![Graph showing total pressure](image)

The total pressure contour as observed from the above figure shows that it is constant almost throughout the entire length of the nozzle with an exception – the walls. There is a decrease in the stagnation pressure near the walls and moving vertically at the exit from the walls to the centre we see that there is a steep rise in the value of stagnation pressure from 5.00e+04 Pa which is the minimum value and is obtained near the walls up to the maximum value of 3.65e+05 Pa which is attained at a little distance from the wall and predominates up to the centre. At the centre of the exit also, the maximum value is attained. The stagnation value has increased from the previous case by 3000 Pa.

**TOTAL TEMPERATURE:**

![Graph showing total temperature](image)

There is a lot of variation in the total temperature plot. In the previous case it was observed that there is an increment in the temperature after the throat and midway from the exit but in this case, there is a difference. Here the temperature tends to increase just after inlet. After this it remains a constant till the exit except near the walls where it increases further and then decreases as we move from the centre to the wall. The value at the inlet is 6.00e+02 K and then increases near the inlet to 6.01e+02 K. At the throat as well as the exit, the value remains the same. The maximum value attained is 6.03e+02 K which is, moving vertically from the wall, some distance from it before which the value steeply increases up to the maximum value and then decreases a little. The maximum value has reduced a little by 1 K considering this case and the previous case of 8°.
The velocity magnitude as usual increases from the inlet till the exit including the throat. As seen from the above figure increase is continuous except near the walls where it is less due to reversals of flow where the fluid almost tends to come at rest as seen from the left panel indicating the values. Thereafter there is increase as we proceed towards the interior. The value at the inlet is $2.18 \times 10^2$ m/s and increases to the throat to $3.92 \times 10^2$ m/s after which it further increases to $8.72 \times 10^2$ m/s at the exit. Excluding the maximum value which has changed a little bit (1 m/s), there is no change in the intermediate values i.e, the values at the inlet and throat in comparison to the previous nozzle observed.

**TURBULENT INTENSITY:**

After taking a look at the above Figure and the one for the previous case, it can be said there is a lot of difference in the contour especially in the region after the throat where there is a steep increase when compared to the last case. The exit value has also changed from $1.57 \times 10^3$ % which was in the previous case to $2.76 \times 10^3$ % which is in the current one. The turbulent intensity at the inlet is $1.26 \times 10^3$ % which decreases a little to $1.04 \times 10^3$ % just after the inlet and remains a constant up to the throat. After the throat there is a sudden increase in the area and hence there is increase in the turbulent intensity. The fluid also does not stabilize here. The maximum turbulent intensity ($5.12 \times 10^3$ %) is obtained near the wall. This is because there is reversal of flow as well as turbulence near the wall.

**MACH 3: - ANGLE = 16°**

Mach Number has been obtained from the above figure. It is found to be 2.92 at the exit, 1.03 at the throat and 0.531 at the inlet. There is also parallel flow observed. There is an increase in the Mach Number value from the previous case of 2.91 to the current case. Also like in the previous nozzle, the flow is Sub-sonic in the inlet region, sonic at the throat and super-sonic at the exit. Reversals of flow cause the irregularities near the walls.

**STATIC PRESSURE:**

Static pressure as usual decreases right from the inlet to the throat and to the exit which was also the scenario in the previous case of nozzle of 12°. The input is at $3.22 \times 10^5$ Pa and decreases to the throat to $1.67 \times 10^5$ Pa and it exits at $1.10 \times 10^4$ Pa. The pattern is similar even though the value has changed by a meager amount.

**TOTAL PRESSURE:**

Total Pressure is constant throughout the nozzle right from the input to the exit except along the walls where it decreases. Stagnation pressure when observed vertically at the exit moving in from the walls, there is minimum value at the walls $5.65 \times 10^4$ Pa and then steeply rises to the maximum of $3.65 \times 10^5$ which continues till the centre. The minimum value has changed but the maximum value is the same when compared to the 12° nozzle.

**TOTAL TEMPERATURE:**

The inlet temperature is $6.00 \times 10^2$ K and it rises just after inlet to $6.01 \times 10^2$ K which again rises to $6.02 \times 10^2$ K at the throat and then there are 2 small patches in the centre just near the throat and 1 small patch at the exit where the temperature remains at $6.02 \times 10^2$ K. At other places with another exception (the walls), the temperature decreases to $6.00 \times 10^2$ K. Moving vertically from the wall to the centre at the exit of the nozzle, minimum temperature of $5.91 \times 10^2$ K is obtained at the walls and then it rapidly increases to the maximum of $6.03 \times 10^2$ K and then again decreases to $6.00 \times 10^2$ K which remains untill it nears the centre where it again rises to $6.01 \times 10^2$ K. The maximum value has remained almost the same whereas a 2 K temperature difference is observed in the minimum value.

**VELOCITY MAGNITUDE:**

The velocity magnitude as usual increases right from the
throat till the exit except the walls where the reversal of flow causes the velocity of the fluid to decrease to almost coming to rest. The value of the velocity at the inlet is 2.18e+02 m/s which increases to the throat to 3.93e+02 m/s and then rapidly increases to attain the maximum value of 8.72e+02 m/s some distance from the exit. The values at the inlet, throat and the outlet have remained more or less constant when compared to the 12° nozzle case.

TURBULENT INTENSITY:-

A pattern similar to that observed in the previous case (of 12°) is also observed here. The pattern is similar but the exit value has increased to 2.82e+03 % from the 2.76e+03 % which was in the 12° nozzle. Inlet is at 1.27e+03 % which then decreases just a little to 1.05e+03 % and remains almost a constant till the throat. After the throat there is a sudden expansion taking place which becomes the prime cause for the turbulence of the liquid. No fluid stabilization is seen here. The maximum value of 5.27e+03 % is obtained near the walls because reversals in the flow increase the turbulence.

IV. CONCLUSION

It is observed that the nozzle which designed for, flow travel along with the direction and at throat above default angle Mach number is 1.03 while at 4° Mach number is 1.01 and at 7.128573° Mach number is 1.02. From Default angle Mach number is increasing up to 2.917 at nozzle exit while for 4° the Mach number at exit is nearly 2.88. At the throat the velocity magnitude around same for all degree of angle and it is 218 m/s. The divergence loss is very low for 12° and 16° for which the nozzle is designed. Near the wall, the Mach number is decreasing for all the nozzles. This is due to the viscosity and turbulence in the fluid. For 4° angle the Mach number at exit is very low compared to other nozzles and the turbulence intensity is very high for 160 and it is nearly 3.045e+05. While when the angle is 80 the mach number at nozzle exit is 2.91 and same as 120° angle but an angle 160° gives the mach number at nozzle exit is 2.92 and it lowest at an angle 40. The degree of angle for conical nozzle can be large as 12 to 18 degree maximum so for maximum thrust we can go with 12° or 16° conical nozzle.

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