Numerical Study of Natural Convection in a Partitioned Square Cavity Filled with Nanofluid

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***Abstract*—In this paper, natural convection heat transfer of a partitioned square cavity filled with Al2O3-water nanofluid is investigated. The vertical left and right walls are considered as the hot and cold walls, respectively and the partition assumed to be adiabatic. The influence of different parameters such as: Rayleigh number (Ra=103-106), distance from the hot wall (d=0.3H-0.7H) and height of partition (h=0.1H-0.3H) and volume fractions of nanoparticles (φ=0-20%) are studied. Results are presented in the form of streamlines, isotherms and average Nusselt number. According to the results, the location and height of the partition are important factors that affect the streamlines and isotherms, especially as the Rayleigh number goes up. It is also found that, the increase in Rayleigh number, increases the average Nusselt number for all the nanofluid volume fractions, but for the studied case, the average Nusselt number is not sensitive to the volume fraction and slight changes occurred. The increment in average Nusselt number is strongly dependent on the partition distance from the hot wall. The highest amount is when the partition's distance is 0.5H.**

***Index Terms*—Nanofluid, Natural Convection, Partition, Square Cavity.**

I. Introduction

The study of natural convection in partitioned fluid-filled square cavities with adiabatic horizontal walls and isothermal vertical walls have attracted great interest among researchers. It is the subject of many engineering applications such as electronic components cooling, transportation, energy storage systems and power stations [1]-[3]. On the other hand, heat transfer in the field of the nanotechnology is the meeting point of the thermal engineering and nanoscale science. Nanofluids are defined as the fluids with nanometer-sized solid-particles suspended in them [4]. The suspended metallic or nonmetallic nanoparticles improve the transport properties and heat transfer characteristics of the conventional fluids such as water, engine oil and ethylene glycol, which have poor thermal properties.

A lot of researches have been conducted to study the performance of the nanofluids. Polidori *et al.* [5] investigated the natural convection heat transfer of newtonian nanofluids (γ -Al_2O_3/water) in a laminar external boundary-layer from the integral formalism approach. It was concluded that natural convection heat transfer is not solely characterized by the nanofluid effective thermal conductivity and that the sensitivity to the viscosity model used seems undeniable and plays a key role in the heat transfer behavior. Khanafer *et al.* [6] studied heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids for various pertinent parameters. They found that the suspended nanoparticles substantially increase the heat transfer rate at any given Grashof number. In addition, the results illustrated that the nanofluid heat transfer rate increases with an increase in the nanoparticles volume fraction. The thermal characteristics of natural convection in a rectangular cavity heated from below with Al_2O_3 nanofluids with Jang and Choi's model for predicting the effective thermal conductivity of nanofluids and various models for the effective viscosity was investigated by Hwang *et al.* [7]. The results showed that water-based Al_2O_3 nanofluids is more stable than base fluid in a rectangular cavity heated from below as the volume fraction of nanoparticles increases, the size of nanoparticles decreases, or the average temperature of nanofluids increases. Abu-nada and chamkha [8] studied the natural convection heat transfer characteristics in a differentially heated enclosure filled with a CuO-EG-Water nanofluid for different variable thermal conductivity and variable viscosity models. According to the results, the effects, the viscosity models are predicted to be more predominant on the behavior of the average Nusselt number than the influence of the thermal conductivity models. In another study, Abu-neda and oztop [9] numerically analyzed the effect of the inclination angle on the natural convection heat transfer and fluid flow in a two-dimensional enclosure filled with Cu nanofluid. They showed that at high Rayleigh numbers, the percentage of the heat transfer enhancement decreased. Furthermore, they proved that the inclination angle can be a control parameter for nanofluid filled enclosure. Eastman *et al.* [10] experimentally observed that Al_2O_3/water and CuO/water with 5% nanoparticle volume fractions increased the thermal conductivity by 29% and 60%, respectively. Sivasankaran *et al.* [11] proved that the type of nanoparticles considered is very important on the convective heat transfer application.

Although a lot of studies have been carried out to investigate the role of nanofluids in cavities, most of them have considered cavities without partition. To our knowledge very little work has been done on partitioned cavities utilizing nanofluids. Anilkumar and Jilani [12] studied the natural convective heat transfer in a partitioned cavity utilizing nano fluids for various pertinent parameters like the solid volume fraction, partition height, Rayleigh numbers

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and aspect ratio of the cavity. The results illustrated that the nano particle solid volume fraction, leads to the increase in nanofluid heat transfer rate.

Therefore, studying the performance of the nanofluid in partitioned cavities needs to be investigated more. The aim of the present paper is to study the Al_2O_3 nanofluid-filled partitioned square cavity. The effects of different locations of the partition as well as the height of the partition is investigated in the different Rayleigh numbers and nanoparticles volume fraction. Moreover, the effect of Rayleigh number, volume fraction and partitions location on the average Nusselt number are studied.

II. Problem description and Mathematical Formulation

A schematic of the two-dimensional cavity is shown in Fig.1. It is a differentially heated square cavity in which an adiabatic vertical partition is attached to it. The hot wall is at T_h at X = 0, and the cold wall is at T_c at X = 1 and the other walls are adiabatic. The fluid in the enclosure is a water-based nanofluid containing Al_2O_3 nanoparticles.

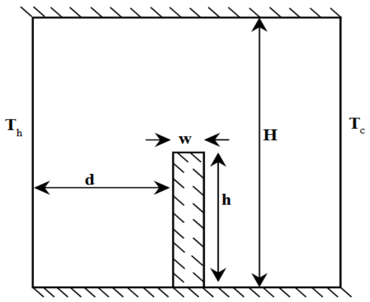

Fig.1. schematic of the partitioned square cavity

It is assumed that the nanofluid is newtonian, incompressible and laminar and the base fluid and the nanoparticles are in a thermal equilibrium state. The thermo-physical properties of the base fluid and the nanoparticles are given in Table. I. Except for the density of the nanofluid which is approximated by the Boussinesq model, the properties of the nanofluid are considered constant.

TABLE. I. Thermophysical Properties of Fluid and Nanoparticles

Properties	Fluid phase (water)	Solid phase (Al_2O_3)
ρ (kg/m^3)	997.1	3970
c_p (J/kg K)	4179	765
β (K^{-1})	2.1×10^{-4}	0.85×10^{-5}

According to the above assumptions, the following formulas are obtained:

Continuity equation:

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \tag{1}$$

X-momentum equation:

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{\partial p}{\partial x^*}+ \Pr_f\left(\frac{C^*_{p,nf}\mu^*_{nf}}{k^*_{nf}}\right)\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right) \tag{2}$$

Y-momentum equation:

$$u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{\partial p}{\partial y^*}+ \Pr_f\left(\frac{C^*_{p,nf}\mu^*_{nf}}{k^*_{nf}}\right)\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right)+ Ra_f\Pr_f\beta^*_{nf}\left(\frac{\rho^*_{nf}C^*_{p,nf}}{k^*_{nf}}\right) \tag{3}$$

Energy equation:

$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\alpha_{nf}\left(\frac{\partial^2 T^*}{\partial x^{*2}}+\frac{\partial^2 T^*}{\partial y^{*2}}\right) \tag{4}$$

The effective density of the nanofluid is given by :

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_p \tag{5}$$

The heat capacitance of the nanofluid is expressed as:

$$\left(\rho c_p\right)_{nf}=(1-\phi)\left(\rho c_p\right)_f+\phi\left(\rho c_p\right)_p \tag{6}$$

The effective thermal conductivity of nanofluid is:

$$\frac{k_{nf}}{k_f}=\frac{k_p+2k_f-2\phi\left(k_f-k_p\right)}{k_p+2k_f+\phi\left(k_f-k_p\right)} \tag{7}$$

The dynamic viscosity of the nanofluid can be given as:

$$\mu_{nf}=\frac{\mu_f}{\left(1-\phi\right)^{2.5}} \tag{8}$$

The thermal expansion coefficient of the nanofluid is expressed by:

$$\left(\rho\beta\right)_{nf}=(1-\phi)\left(\rho\beta\right)_f+\phi\left(\rho\beta\right)_p \tag{9}$$

The non-dimensional parameters used for the equations are:

$$x^*=\frac{x}{L}\ ,\ y^*=\frac{y}{L}\ ,\ u^*=\frac{uL}{\alpha}\ ,\ v^*=\frac{vL}{\alpha}\ ,\ T^*=\frac{T-T_c}{T_H-T_c}\ ,$$

$$\rho^*=\frac{\rho_n}{\rho_f},\ \mu^*=\frac{\mu_n}{\mu_f},\ k^*=\frac{k_n}{k_f},\ cp^*=\frac{c_{p,n}}{c_{p,f}}\ ,\ \beta^*=\frac{\beta_n}{\beta_f}$$

The Rayleigh and Prandtl numbers are defined by:

$$Ra_f=\frac{g\beta_f\left(T_h-T_c\right)}{\nu_f\alpha_f} \tag{10}$$

$$\Pr_f=\frac{\nu_f}{\alpha_f} \tag{11}$$

The boundary conditions are as follows:

$$u^* = v^* = \frac{\partial T^*}{\partial y^*} = 0 \quad \text{at} \quad 0 \le x^* \le 1 \;\text{ and }\; y^* = 0,1$$

$$u^* = v^* = 0, \quad T^* = 1 \;\text{ at }\; x^* = 0 \;\text{ and }\; 0 \le y^* \le 1$$

$$u^* = v^* = 0, \quad T^* = 0 \;\text{ at }\; x^* = 0 \;\text{ and }\; 0 \le y^* \le 1$$

The local Nusslet number is defined by:

$$Nu = \frac{hL}{k_f} = -\left(\frac{k_{nf}}{k_f}\right)\frac{\partial T}{\partial x} \tag{12}$$

And the average *Nusselt number is:*

$$\overline{Nu} = \int_0^1 Nu(y)dy \tag{13}$$

III. Numerical Procedure

The governing equations as well as boundary conditions are solved by finite volume method. The SIMPLE algorithm is used for handling the pressure velocity coupling. In order to determine the proper grid size for this study, the grid independence study is conducted using different grid resolutions. Table. II presents the influence of the number of grid points on Nu_{avg}.

TABLE. II. Grid Independence Study for Cavity without Partition Ra=10^5

Mesh size	Nu_{avg}	Error (%)
31×31	4.667	
41×41	4.6	1.435
51×51	4.567	0.7174
61×61	4.549	0.394
71×71	4.538	0.241
81×81	4.531	0.154
91×91	4.526	0.11
101×101	4.523	0.066

It is observed that a 91×91 grid, is accurate enough to be chosen as the grid for all the calculations. For validation, the results of the numerical results for Nu_{avg} in a square cavity are compared with the results of others. The comparison is shown in Table. III which shows a very good agreement.

TABLE. III Comparison Between Present Work and other Published Data

Ra	10^4	10^5	10^6
Present work	2.241	4.526	8.919
de Vahl Davis [13]	2.243	4.519	8.8
House *et a.l* [14]	2.254	4.561	8.923
Merrikh and Lage [15]	2.244	4.536	8.86
Kalita *et al.* [16]	2.245	4.522	8.829
Braga *et al.* [17]	2.249	4.575	8.918
Tric *et al.* [18]	2.245	4.522	8.825

IV. Results and Discussion

A numerical study is performed to investigate the natural convection in a nanofluid-filled (Al_2O_3-water) partitioned square cavity with isothermal vertical walls and adiabatic horizontal walls. The range of the Rayleigh number is $10^3 \le Ra \le 10^6$, the volume fraction ϕ of the particles varied between $0 \le \phi \le 20\%$, the different locations of the partition $(d = 0.3, 0.5, 0.7H)$ and different heights of the partition $(h = 0.1, 0.3, 0.5H)$. All the calculations are performed at a constant partition width (i.e. $w = 0.1H$).

To study the effects of the partition location, streamlines and isotherm are shown in Fig.2 at $Ra = 10^5$, fixed partition height $(h = 0.5)$ and for $\phi = 10\%$. The plots are arranged from left to right with the increase in distance from the hot wall and at $\phi = 10\%$. For isotherms, the contour level increments are kept at 0.1. According to the streamlines, there are two elliptic vortexes when the distance from the hot wall is d=0.3 but by increasing the distance from the left wall, the shape of the vortex changes, and it becomes a unit one, and it moves toward the hot wall. The plots show that the isotherms are skewd. As the partition approaches the cold wall, its heat transfer rate increases.

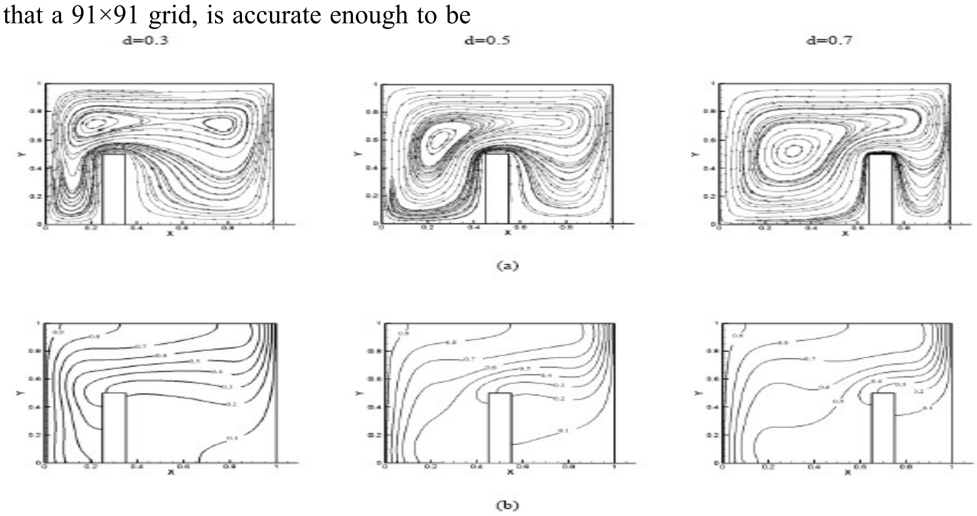

Fig.2. effect of partition location at Ra=10^5, $\phi = 10\%$ and $h = 0.5$ (a) streamlines, (b) isotherms

Fig.3 shows the effect of the partition height increase in the form of streamlines and isotherms for $Ra = 10^5$, $d = 0.5$ and $\phi = 10\%$. According to the streamlines, when the height of the partition is little, there are two elliptic vortexes in the middle of the cavity. As the height becomes larger, the vortex losses its elliptical shape, and it moves to the hot wall. Moreover, in the little partition, the isotherms treat as horizontal and uniform lines and become vertical only inside the thermal boundary layers at the vertical walls. However, as the height increases, the equality of the lines alters, and they become skewed lines. The rate of heat transfer decreases with the rise of the partition height.

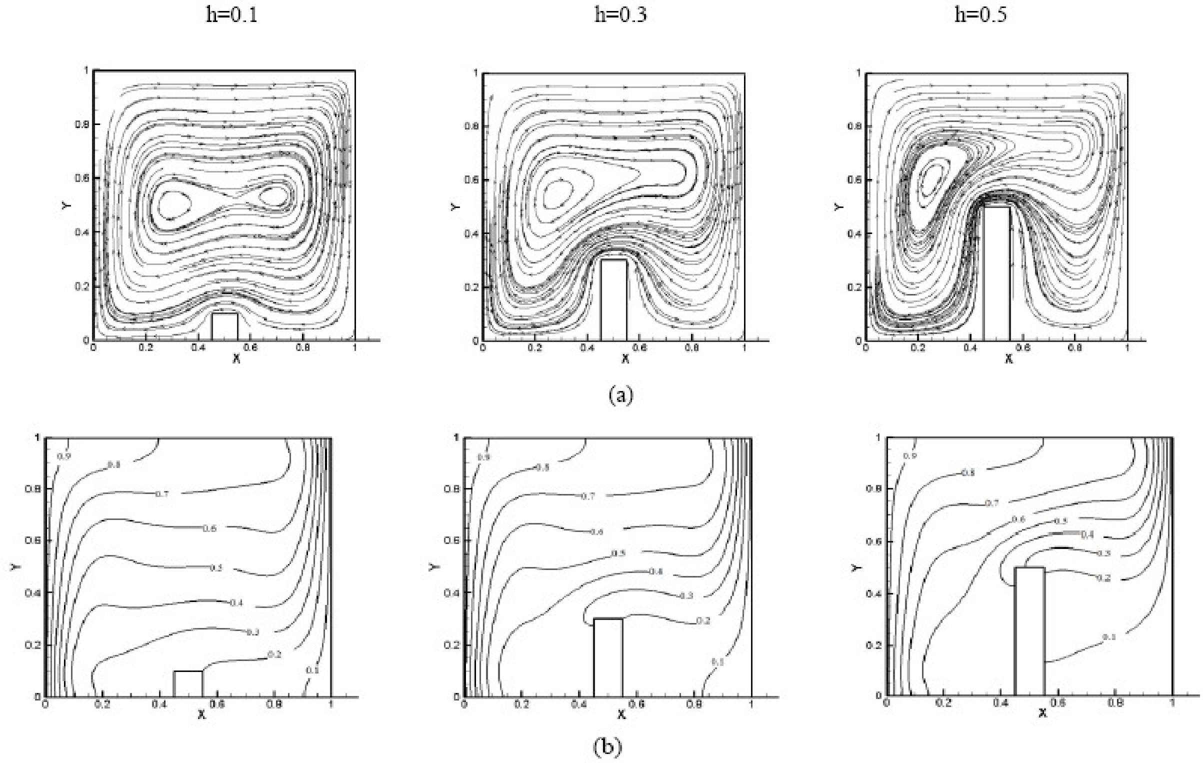

Fig.3. effect of partition height at Ra=10^5, $\phi = 10\%$ and $h = 0.5$ (a) streamlines, (b) isotherm

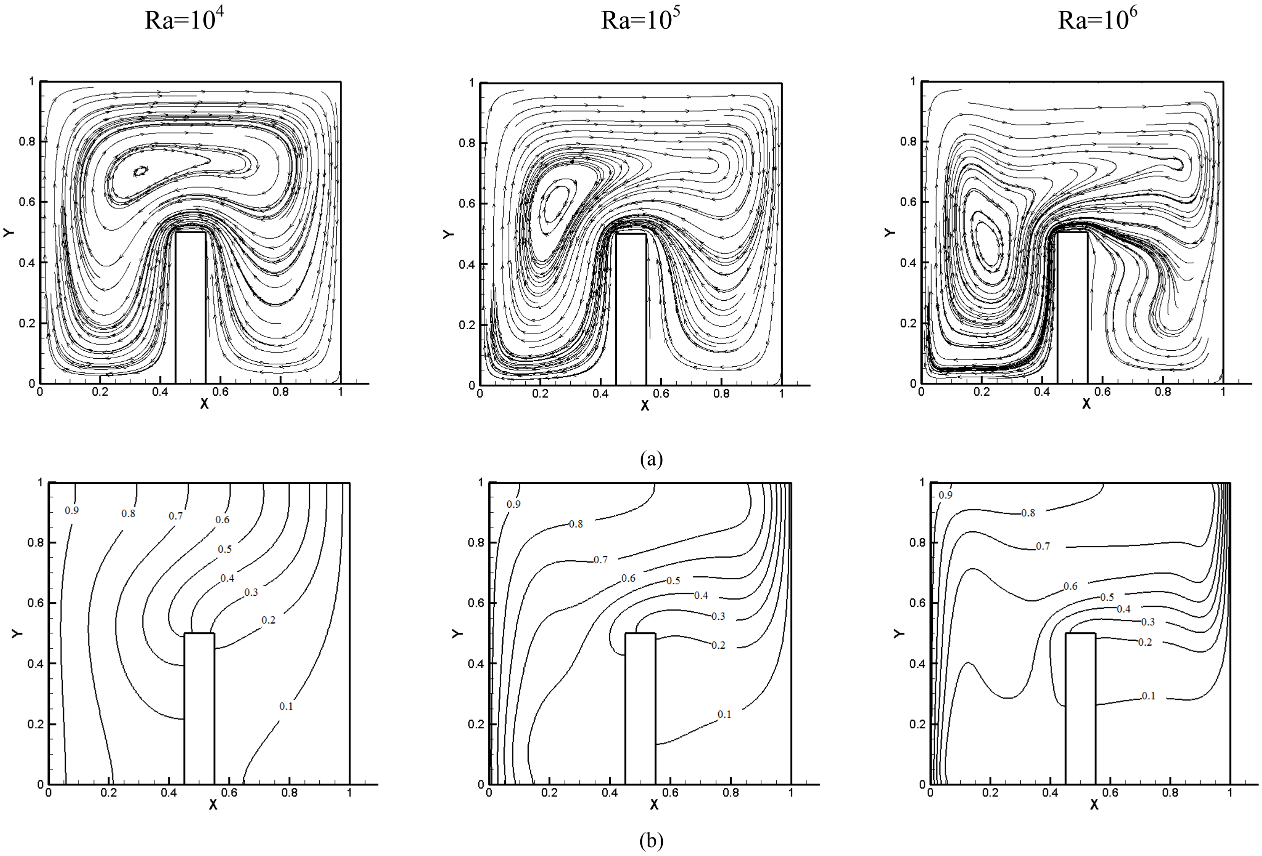

Fig.4. effect of Rayleigh number at $D = 0.5$, $\phi = 10\%$ and $h = 0.5$ (a) streamlines, (b) isotherms

Fig.4 displays the streamlines and isotherms for different Rayleigh numbers varying from 10^3-10^6 for a centered partition with $h = 0.5$ and for $\phi = 10\%$. As the Rayleigh number rises, the convection in the cavity overcomes conduction and therefore, isotherms becomes skewed. At Ra=10^6, the degree of the distortion from the conduction is a lot and so the lines tend to be horizontal.

Fig.5 indicates the variation of the average Nusselt number at the hot wall with the volume fraction for different Rayleigh numbers. With the increase in Rayleigh number, the average Nusselt number increases for all the volume fractions. The graphs indicate that the average Nusselt number does not always increase with the use of nanofluid. This result has been discussed before by Ho et al. [19].

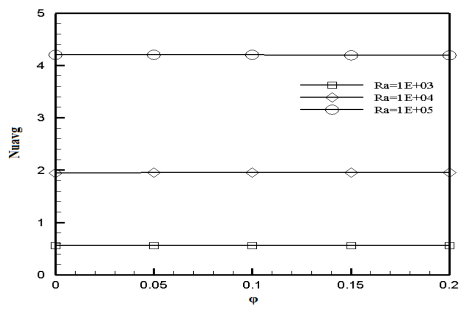

Fig.5. effect of Rayleigh number on the average Nusselt number

The effect of the volume fraction on the average Nusselt number for different distances of partition from the hot wall at Ra=10^5 is depicted in Fig.6. It is found that when the partition is near the hot wall, the average Nusselt number decreases with the increasing volume fraction. However, as it approaches the cold wall, the average Nusselt number increases with ϕ. Furthermore, it is found that the Nu_{avg} is maximum for the case in which partition is placed at the center.

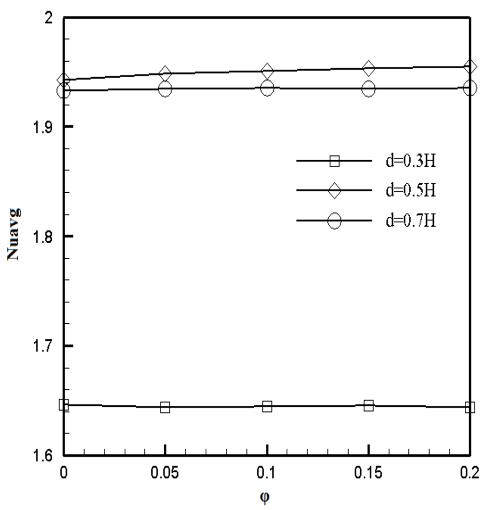

Fig.6. effect of partition distance from hot wall on the average Nusselt number

V. Conclusions

A differentially heated square cavity filled with Al_2O_3-water nanofluid is investigated in this paper. A vertical adiabatic partition is attached to it. Finite volume method is used to solve the problem numerically. Validation is conducted by comparing with different previously published data. The variation of different parameters, including Rayleigh number, partition distance from the hot wall, partition height and the volume fraction of the nanoparticles is studied. The main conclusions from this study at the defined ranges are as follows:

- At a fixed Rayleigh number, volume fraction and partition height, as the partition approaches the cold wall, its heat transfer rate increases.
- Increase in the height of the partition, decreases the heat transfer rate.
- Increasing Rayleigh number, leads to the increase of the average Nusselt number.
- The average Nusselt number is maximum when the partition is placed at the center ($d = 0.5$).

Appendix

The algorithm in finite volume method, includes three steps:

1. Calculation of the governing equations integral on all control volumes.
2. Discretization, which converts the equations to a system of algebraic equations.
3. Solve the equations with an iteration method.

Discretizing the governing equations by the finite volume method, leads to the following equation:

$$a_P \Phi_P = \sum a_{nb} \Phi_{nb} + S_U \tag{A.1}$$

in which Φ is the term standing for u and v, S is the source term and nb is the neighborhood grid of P.

The u-momentum and v-momentum equations for the control volume are expressed as:

$$a_e u_e = \sum a_{nb} u_{nb} + (P_P - P_E) A_e + S_U \tag{A.2}$$

$$a_n v_n = \sum a_{nb} v_{nb} + (P_P - P_N) A_n + S_U \tag{A.3}$$

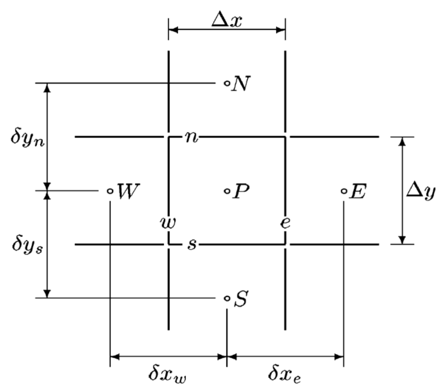

Fig.A.1. control volume

By solving equations (A.2) and (A.3), pressure distribution P^* can be evaluated from velocities u^* and v^*:

$$a_e u_e^* = \sum a_{nb} u_{nb}^* + (P_P^* - P_E^*) A_e + S_U \tag{A.4}$$

$$a_n v_n^* = \sum a_{nb} v_{nb}^* + (P_P^* - P_N^*) A_n + S_U \tag{A.5}$$

However, velocities obtained from equations (A.4) and (A.5) would not satisfy the mass conservation condition for each control volume. Therefore, $u^{'}, v^{'}$ and $P^{'}$ are introduced as the correction terms for velocity and pressure, respectively.

$$u = u^{'} + u^{*} \tag{A.6}$$

$$v = v^{'} + v^{*} \tag{A.7}$$

$$P = P^{'} + P^{*} \tag{A.8}$$

u, v and P are the terms that satisfy both the mass and momentum constraints. Substituting equations (A.4) into (A.2) and (A.5) into (A.3) results in:

$$a_e u_e^{'} = \sum a_{nb} u_{nb}^{'} + (P_P^{'} - P_E^{'}) A_e \tag{A.9}$$

$$a_n v_n^{'} = \sum a_{nb} v_{nb}^{'} + (P_P^{'} - P_N^{'}) A_n \tag{A.10}$$

According to SIMPLE method:

$$u^{'}_e = u^{'}_{e1} + u^{'}_{e2} \tag{A.11}$$

$$v^{'}_n = v^{'}_{n1} + v^{'}_{n2} \tag{A.12}$$

$$P^{'} = P^{'}_1 + P^{'}_2 \tag{A.13}$$

where

$$u^{'}_{e1} = (P^{'}_{P1} - P_{E1}^{'}) \frac{A_e}{a_e} \tag{A.14}$$

$$u^{'}_{e2} = \frac{\sum a_{nb} u_{nb}}{a_e} + (P^{'}_{P2} - P_{E2}^{'}) \frac{A_e}{a_e} \tag{A.15}$$

The value of u'_{e2} would be zero because of the first-order approximation. Therefore,

$$u_e = u^{*}_e + d_e (P^{'}_P - P^{'}_E) \tag{A.16}$$

$$v_n = v^{*}_n + d_n (P^{'}_P - P^{'}_N) \tag{A.17}$$

in which d_e and d_n are defined as

$$d_e = \frac{A_e}{a_e}, \; d_n = \frac{A_n}{a_n} \tag{A.18}$$

By substituting equations (A.16) and (A.17) into the continuity equation, the pressure correction term is derived as:

$$a_P P^{'}_P = a_N P^{'}_N + a_E P^{'}_E + a_S P^{'}_S + a_W P^{'}_W + S_U \tag{A.19}$$

where

$$S_U = (\rho u^{*} A)_N + (\rho u^{*} A)_E + (\rho u^{*} A)_S + (\rho u^{*} A)_W \tag{A.20}$$

Nomenclature

c_p	specific heat at constant pressure (J/kg K)
d	distance of partition from hot wall (m)
g	gravitational acceleration (m /s^2)
H	cavity height (m)
h	partition height (m)
k	thermal conductivity (W/m K)
p	pressure (N/m^2)
Pr	Prandtl number = υ_f/α_f
Ra	Rayleigh number = $g\beta\Delta TH^3/\alpha\upsilon$
T	temperature (K)
u,v	x and y components of velocity (m/s)
w	partition width (m)
x,y	Cartesian coordinates (m)

Greek symbols

α	thermal diffusivity (m^2/s)
β	thermal expansion coefficient (K^{-1})
φ	nanoparticle volume fraction
υ	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)
μ	dynamic viscosity (m^2/s)

Subscripts

avg	average
c	cold
f	fluid
h	hot
nf	nanofluid
p	particle

Subscript

*	non-dimensional form

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