Modified Relay Tuning Method for Level Control Model

S.A. Misal, Vivek S Sathe, R. W. Gaikwad, and Dhirendra

***Abstract*—The liquid level control unit approximated by the first order plus time delay (FOPTD) model was considered and the proposed modified relay tuning method was applied to calculate the corrected ultimate gain (Ku). The proposed method includes the correction factor in equation for calculation of ultimate gain (Ku). The effect of ratio of time delay to time constant was observed and it was found that the error in calculation of ultimate gain by proposed method has been reduced than that of conventional method. The corrected Ku by proposed method was used to estimate PID parameters by Tyreus-Luyben method. The closed loop response was obtained using optimum values of PID parameters and compared with that of conventional method. It was found that the Integral Square Error (ISE) for conventional and proposed method is 18 and 11 respectively and hence the proposed method gives better performance.**

***Index Terms*—Modified Relay Tuning, FOPTD, Simulink.**

I. Introduction

Control plays a key role in the operation of chemical plants with respect to economical performance, safety and operability. In a typical chemical plant there are hundreds of PID feedback loops. They are often poorly tuned because the choice of PID controller parameters requires professional knowledge by the user. One of the most common approaches to tune a controller automatically is to connect a relay as a feedback controller to the process during tuning. Astrom [1] have suggested the use of an ideal (on–off) relay to generate a sustained oscillation of the controlled variable and to get the ultimate gain (Ku) and the ultimate frequency (ω_u) directly from the relay experiment. The relay feedback method has become very popular because, it is time efficient as compared to the conventional method. The amplitude (a) and the period of oscillation (Pu) are noted from the sustained oscillation of the system output. The ultimate gain (Ku) and ultimate frequency (ω_u) are calculated from the principal harmonics approximation as given by equation;

$$Ku = \frac{4h}{\pi a} \tag{1}$$

$$\omega_u = \frac{2\pi}{P_u} \tag{2}$$

The use of relay testing for identifying a transfer function model has suggested by W L Luyben [5]. Using Ku and ω_u in the phase angle and amplitude criteria for an unstable FOPTD model, the following two equations relating three model parameters are obtained

$$\frac{K_u K_P}{\left(1+\tau^2\omega_u^2\right)^{0.5}} = 1 \tag{3}$$

$$D\omega_u - \tan^{-1}(\tau\omega_u) = 0 \tag{4}$$

Since only Ku and ω_u are available, additional information such as the steady state gain, or the time delay should be a known priori in order to fit a typical transfer function model such as unstable FOPTD. The above equations assume that, the higher order harmonics are neglected**.**

A method of identifying a FOPTD unstable model based on the shape of the response of the process using a symmetric relay has been proposed by Thyagarajan and Yu [9]. In this method, the output response is aligned with the input response by shifting to the left. Then, the time to peak amplitude, the peak amplitude and the period of oscillation are noted. The time delay is considered as the time to the peak value. From the derived analytical expression of the process output response of an unstable FOPTD system for a symmetric relay input, the time constant and gain are calculated as;

$$\tau = \frac{\frac{P_u}{2}}{\ln\left(\frac{1}{2e^{D/\tau}-1}\right)} \tag{5}$$

$$K_p = \frac{a}{h\left(e^{D/\tau}-1\right)} \tag{6}$$

It is to be noted that, for higher order systems, the recorded time to peak value from the response will not match with that of the actual time delay of the process. Then it was reported by Li, Eskinat and Luyben [4] that the models identified by the symmetry relay auto tune method gives error as high as 27 to −18% in the value of Ku for stable FOPTD systems.

Recently Srinivasan and Chidambaram [8] have proposed a method of considering higher order harmonics, to explain the reported error of 27 to −18% in Ku calculations for stable systems. An improved method by incorporating the higher order harmonics has been proposed by Sathe Vivek, M. Chidambaram [7] to explain the error in the *Ku* calculation. The relay equation is given as;

$$y(t) = a\left[1+\frac{1}{9}+\frac{1}{25}+\frac{1}{49}+...\right] \tag{7}$$

In this paper, a correction factor is applied to calculate the corrected ultimate gain (Ku) and reduce the error in Ku calculation. This corrected value was used to estimate the optimum PID parameters.

II. Materials and Method

The liquid level control unit was considered as shown in Figure 1 which consist of a tanks placed on a rig where the input flow is manipulated by means of pump voltage and thus

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the liquid level is controlled.

The model equation for deriving the transfer function was assumed to be the simplest nonlinear model of the tank system relating the water level *h with* the voltage *u* applied to the pump as;

$$\frac{dh(t)}{dt} = -\frac{a_1}{A}\left(2gh(t)\right)^{0.5} + \eta . u(t) \qquad (8)$$

where h – Water level in tank, a_1 – tank outlet area, A – cross-sectional area of the tanks, *g* – Gravitational constant, *ή*-Constant relating the control voltage with the water flows from the pump.

Linearization of above model gives:

$$\frac{d\Delta h(t)}{dt} = \frac{d}{dh}\left[-\frac{a_1}{A}\left(2gh(t)\right)^{0.5} + \eta . \Delta u(t)\right]$$

Laplace transformation of above equation obtained is;

$$s\Delta H(s) = -\left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0}\Delta H(s) + \eta . \Delta u(t)$$

Rearranging the above equation to get the transfer function;

$$\frac{\Delta H(s)}{\Delta U(s)} = \frac{\eta}{s + \left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0}} \qquad (9)$$

For given system parameters, the obtained transfer function is FOPTD model;

$$\frac{\Delta H(s)}{\Delta U(s)} = \frac{0.058}{0.44s + 1} e^{-0.3s} \qquad (10)$$

where time constant (τ) is 0.44; process gain (k_p) is 0.058 and time delay (D) is 0.3.

Then the Simulink diagram for relay tuning was prepared as shown in Figure 2. The relay experiments were carried out for relay height as 1. Four different values of the ratio of time delay to time constant i.e. τ/D were considered as shown in Table 1. The amplitude (a), period of oscillations (Pu) was noted. The corrected ultimate gain was calculated using proposed modified equation using equation (1) as;

$$K_u = \frac{4h}{\pi a} + k \qquad (11)$$

where k is correction factor & approximated as;

k=25% (Relay height / observed process amplitude) ;

Hence equation (11) becomes;

$$K_u = \frac{4h}{\pi a} + 0.25\frac{h}{a} \qquad (12)$$

The error in Ku calculation for different values of τ/D was estimated using conventional and proposed method. The PID controller tuning parameters were calculated using Ziegler Nichols [10] method for conventional relay method. Tyreus-Luyben method [6] was used for modified relay method as shown in Table 2. Then using these optimum values of PID parameters, the closed loop response was studied and it was compared with that of the conventional method. Integral Square Error (ISE) was calculated for both the experiments

III. Result & Discussion

For relay experiment, relay height (h) as 1 was considered. The experiments were conducted and the corrected ultimate gain (Ku) was calculated using equation (10). The actual value of Ku by amplitude criteria, the calculated values of Ku by conventional and improved relay method are shown in Table 1. Thus the error in calculation of Ku for given process with D/τ as 0.2, obtained by conventional method found to be 16% and that by improved relay method has been reduced up to 1%. The Simulink diagram for relay experiment, relay response and process response are shown in Figure 2-4 respectively. The comparison of % Error for various values of D/τ is shown in Figure 5. The PID parameters were calculated using Tyreus-Luyben method as shown in Table 3. The Simulink diagram for PID experiment is shown in Figure 6. Using these optimum values, the closed loop response was obtained and it was compared with that of conventional method as shown in Figure 7.

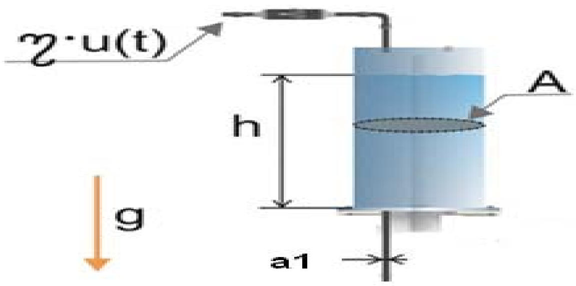

Fig. 1 Level Control system

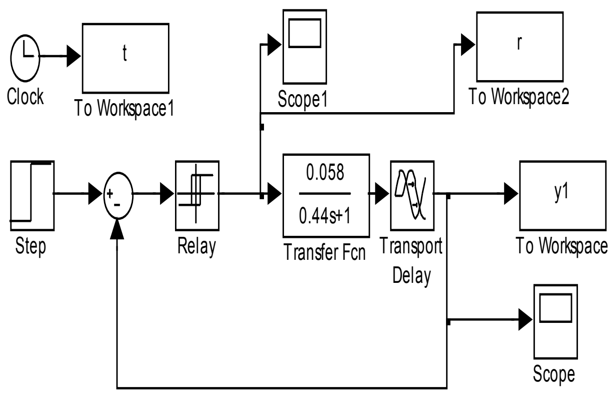

Fig. 2 Simulink Diagram for Relay Experiment

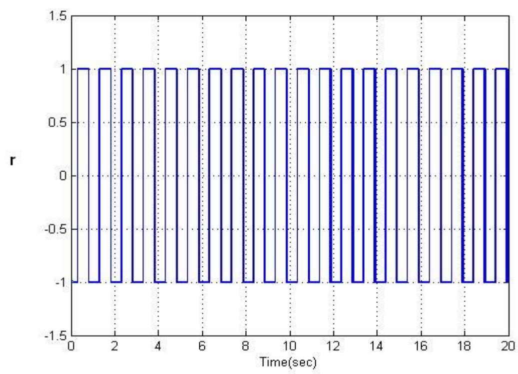

Fig. 3 Relay Response.

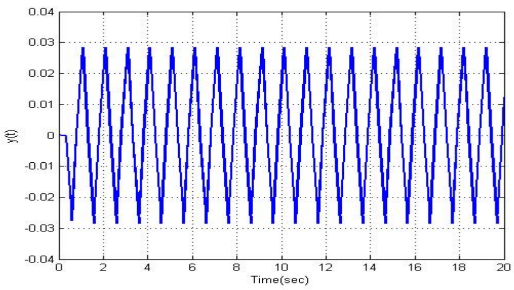

Fig.4 Process Response

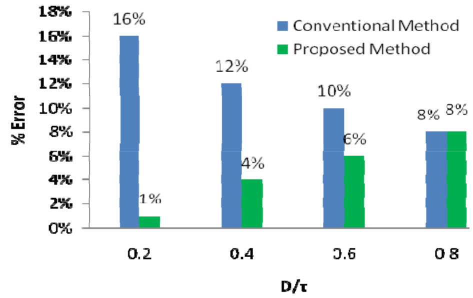

Fig. 5 Comparison of % Error for various values of D/τ

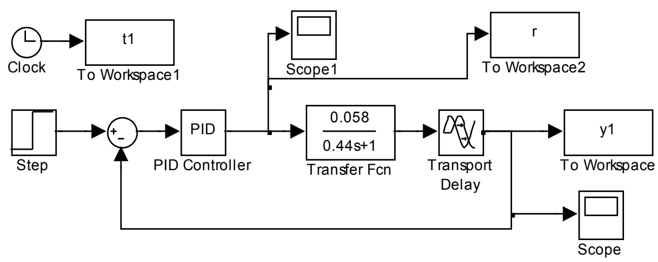

Fig. 6 Simulink Diagram for PID Control

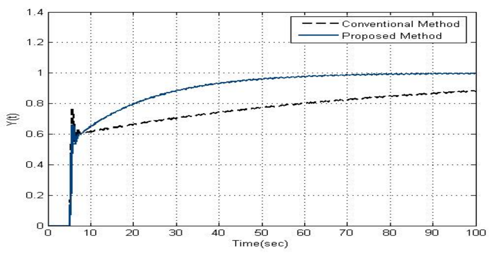

Fig. 7 Closed Loop Response with optimum values of PID Parameters.

TABLE 1 COMPARISON OF METHODS FOR PROCESS MODEL

D/τ	Ku Actual	Ku (Calculated)		% Error	
		Conventional Method	Proposed Method	Conventional Method	Proposed Method
0.2	50	42	50.5	-16%	1%
0.4	50	44	52	-12%	4%
0.6	50	45	53	-10%	6%
0.8	50	46	54	-8%	8%

TABLE 2: PID CONTROLLER PARAMETER SETTINGS

Method	Kc	τ_I	τ_D
Ziegler-Nichols Method	Ku/1.7	Pu/2	Pu/8
Tyreus-Luyben Method	Ku/2.2	2.2Pu	Pu/6.3

TABLE 3 CALCULATED PID CONTROLLER PARAMETER

Method	Kc	τ_I	τ_D
Conventional Method	24.7	0.55	0.13
Proposed Method	20	2	0.17

IV. CONCLUSION

In this study, a modified Relay-tuning PID control scheme for the level control model is presented. By using this scheme, the parameters are optimally and robustly adjusted with respect to the system dynamics. This technique is found to be more effective than conventional tuning methods. This method can be easily extended to multi input and multi-output systems from basic single-input and single-output systems. The simple structure, robustness and ease of computation of the proposed method make it very attractive for real time implementation for control of given process. It was found that the error in calculation of ultimate gain by conventional method was in the rage 8% to 16% and that has been reduced to range of 1% to 8% using proposed method and hence the proposed method gives better performance. The Integral Square Error (ISE) for conventional and proposed method is 18 and 11 respectively.

Nomenclature;
Pu-Period of oscillation
Ku-Ultimate gain
ω_u -Frequency of oscillation
h-Relay height
a-amplitude
N -No. of harmonics in relay equation
kc-Controller gain
τ_I-Integral time, sec
τ_D-Derivative time, sec
K-Correction factor

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