

# Identification and Control of Unstable Biochemical Reactor

V.Rajinikanth, K.Latha

**Abstract**— The work presented in this paper deals with the process of transfer function identification for unstable biochemical reactor. It is a closed loop test and the output from the system for a single step change with a Proportional (P) controller can be used to estimate process model parameters like steady-state gain ( $K$ ), time constant ( $\tau$ ) and time delay ( $d$ ). In this paper, we discuss the role of damping ratio ( $\zeta$ ) in closed loop identification with a Proportional controller. A Particle Swarm Optimization (PSO) method based PID controller tuning is also proposed for the identified transfer function model. To validate the control performance of the proposed method, it is compared with an IMC controller. The simulation result shows that the PSO algorithm can perform well in the unstable PID control system design and it improves the performance of the process in terms of time domain specifications, set point tracking, disturbance rejection, error minimization and also provides an optimum stability.

**Index Terms**— Step response test, PSO, PID controller, Unstable systems.

## I. INTRODUCTION

System identification is one of the major research areas in the field of process control and the parametric identification method can be carried out with step response test or relay feedback test. Real processes are very complicated and determining precise model is not possible. Usually, the real processes are represented by an approximate low order plus time delay models, like first order plus time delay (FOPTD) model and a second order plus time delay (SOPTD) model [1]. An unstable FOPDT process under relay feedback produces limit cycle when the ratio of time delay to unstable time constant ( $d/\tau$ ) is less than 0.693 [1, 8]. A modified relay feedback should be used when  $d/\tau \leq 0.1$  and the  $d/\tau$  ratio can be extended up to 1 by providing an inner feedback proportional controller during the auto tuning test [8]. Due to the  $d/\tau$  limitation of relay feedback test, a step response method is most commonly used for system identification in process industries. For unstable systems, open loop method cannot be used and it is sensitive to disturbances. So the closed loop method is preferred to identify the approximated FOPTD model of the process. PI/PID based step response test was proposed by most of the

researchers to overcome the offset due to the P controller and the PI controller cannot be used when  $d/\tau \geq 0.7$  [1, 5]. To identify the unstable FOPTD model by using the PID controller, computation time required is more than the P/PI controller based model estimation. The parameter to be tuned in P controller based method is only the proportional gain ' $K_c$ ' and it is very simple than the PI/PID based step response test.

In process industries more than 95% of the controllers are PI/PID (Proportional plus Integral / Proportional plus Integral plus Derivative) type. There are many tuning formulae available for PI/PID controllers in the literature for unstable processes [1,5,6,7,8,9]. The most critical step in application of PID controller is parameters tuning. So it is necessary to implement biologically inspired intelligent PID controller parameter optimization method.

Particle Swarm Optimization is a population based stochastic optimization technique first introduced by Kennedy and Ebert in 1995 [10], inspired by social behaviour of bird flocking or fish schooling, and it is widely used in engineering applications due to its high computational efficiency, easy implementation and stable convergence [11]. PSO algorithm is easy to implement and there are few parameters to adjust and has been successfully applied in many areas such as function optimization, fuzzy gain scheduling, PID Auto-tuning and fractional order PID controller design [12,13,14,15].

In this paper, a simple method is proposed to estimate the model parameters of unstable system using a P controller with desired damping ratio ( $\zeta$ ). The proposed method is generally applicable to unstable processes with wide range of  $d/\tau$  ratio. We also proposed a PSO based PID controller for the biochemical reactor and the controller performance is compared with a modified Internal Model Controller (IMC) controller.

## II. BIOCHEMICAL REACTOR

The biochemical reactor is an essential unit operation in a wide variety of biotechnological processes. Biochemical reactors are used to produce a large number of intermediate and final products, including medical products, food, beverages, and industrial solvents [2, 3]. For biochemical reactors, unstructured models are mainly used for its simplicity. The following unstructured models can describe a variety of bioreactors and schematic of a biochemical reactor is as shown in Fig.1. The modelling equations are;

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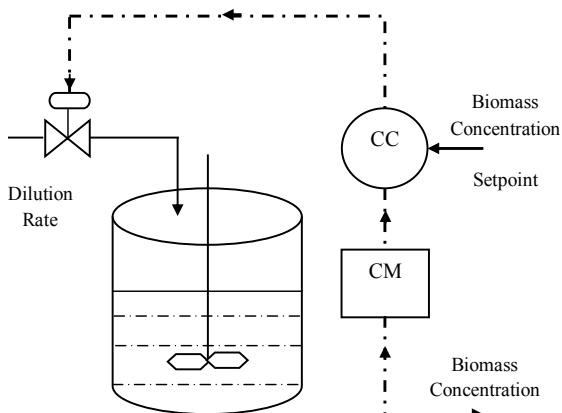


Fig. 1 Biochemical Reactor

$$\frac{dx_1}{dt} = (\mu - D)x_1 \quad (1)$$

$$\frac{dx_2}{dt} = D(x_{2f} - x_2) - \frac{\mu x_1}{Y} \quad (2)$$

$$\mu = \frac{\mu_{max}x_2}{k_m + x_2 + k_1x_2^2} \text{ Substrate Inhibition} \quad (3)$$

Where  $x_1$  is biomass (cell) concentration,  $x_2$  is substrate concentration,  $D$  is the dilution rate,  $x_{2f}$  is substrate feed concentration. For substrate inhibition model, the following parameters are considered.  
 $\mu_{max} = 0.53 \text{ hr}^{-1}$ ,  $k_m = 0.12 \text{ g/l}$ ,  $k_1 = 0.4545 \text{ l/g}$ ,  $Y = 0.4$ . The steady state dilution rate is  $D_s = 0.3 \text{ h}^{-1}$  (the residence time is 3.33 h) and the feed substrate concentration is  $x_{2fs} = 4.0 \text{ g/l}$ . The nonlinear process has the three steady state operating points for a dilution rate of 0.3  $\text{h}^{-1}$ . For the unstable operating region (equilibrium 2 – nontrivial) biomass concentration  $x_{1s} = 0.995103$  and substrate concentration  $x_{2s} = 1.512243$  are considered. The dilution rate is taken as the manipulated variable in order to control the cell mass concentration at the unstable steady state. A delay of 1h is considered in the measurement of  $x_1$ [2, 6, 7].  
For the given condition of the unstable operating point, the local linearized first order plus time delay transfer function model for the unstable bioreactor is

$$\frac{\Delta X(s)}{\Delta D(s)} = \frac{K \exp^{-ds}}{(\tau s - 1)} = \frac{-5.89 \exp^{-1s}}{(5.86s - 1)} \quad (4)$$

### III. STEP RESPONSE TEST

It is a closed loop test and for controller design purposes an unstable FOPTD is taken as the system [1].

$$\text{Let the true process } G(s) = K \exp^{-ds} / (\tau s - 1) \quad (5)$$

The closed loop transfer function of the system with a P controller is given by

$$Y(s) / R(s) = G_p(s) G_c(s) / (1 - G_p(s) G_c(s)) \quad (6)$$

$$= K \exp^{-ds} K_c / [(\tau s - 1) + K \exp^{-ds} K_c] \quad (7)$$

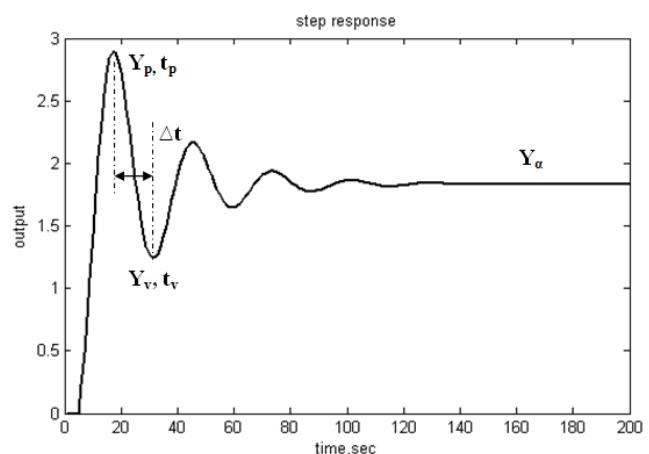


Fig.2. Step response of unstable FOPTD system with P controller

Replacing  $\exp^{-ds}$  with its first order Pade's approximation, and solving Eq. (3), the following equations are obtained.

$$K^1 = K K_c \quad (8)$$

$$\zeta = -\ln(v) / \sqrt{\Pi^2 + \{\ln(v)\}^2} \quad (9)$$

$$V = (Y_a - Y_v) / (Y_p - Y_a) \quad (10)$$

Here  $Y_p$ ,  $Y_v$  and  $Y_a$  are first peak, first valley and the final value of response (Fig.2). The second peak and the second valley are not considered.

The parameters of the process can be identified from,

$$K = Y_a / [K_c (Y_a - 1)] \quad (11)$$

$$T = \Delta t P_1 P_2 / \Pi \quad (12)$$

$$d_c = \Delta t 2 P_1 / (P_2 \Pi) \quad (13)$$

$$\Delta t = t_v - t_p \quad (14)$$

$$P_1 = \sqrt{(1 - \zeta^2)(K^1 - 1)} \quad (15)$$

$$P_2 = \zeta \sqrt{(K^1 - 1)} + \sqrt{\zeta^2 (K^1 - 1) + (K^1 + 1)} \quad (16)$$

By using above Equations (Eq. (8) to Eq. (16)) the unstable FOPDT model of the process can be obtained [1].

## VI. CONTROLLER DESIGN

### A. PID Controller

The flexibility and robustness of the PID controller makes it widely applied in many applications. The continuous control law of PID controller is

$$u(t) = K_p e(t) + K_i \int_0^t e(t) d(t) + K_d \frac{d}{dt} e(t) \quad (17)$$

Where  $e(t)$  is the error signal between the setpoint and actual output,  $u(t)$  is the controller output and  $K_p$ ,  $K_i$ ,  $K_d$  are the PID controller gains. A basic PID controller directly operates on the error signal and this may produce a large overshoot in the process response due to the proportional and derivative kick. The process is unstable and to

overcome the effect of proportional and derivative kick, a modified PID structure known as I-PD is considered (Fig 2). In I-PD structure, the integral term responds based on the error and the P+D terms works based on the measured process output [4].

I-PD formula is

$$u(t) = \frac{K_i}{s} e(t) - [K_p + K_d s] Y(s) \quad (18)$$

### B. PSO algorithm based PID

The PSO algorithm attempts to mimic the natural process of group communication of individual knowledge, to achieve some optimum property.

The swarm is initialized with a population of random solutions. Each particle in the swarm is a different possible set of the unknown parameters to be optimized. Representing a point in the solution space, each particle adjusts its flying toward a potential area according to its own flying experience and shares social information among particles. The goal is to efficiently search the solution space by swarming the particles toward the best fitting solution encountered in previous iterations with the intent of encountering better solutions through the course of the process and eventually converging on a single minimum error. At the beginning, each particle of the population is scattered randomly throughout the entire search space. Under the guidance of the performance criterion, particles in their flies dynamically adjust their velocities according to their own flying experience and their companions flying experience. Each particle remembers its best position obtained so far, which is denoted *pbest*. It also receives the globally best position achieved by any particle in the population, which is denoted as *gbest*. The updated velocity of each particle can be calculated using the present velocity and the distances from *pbest* and *gbest* [14].

The mathematical expression for Velocity update

$$V_i^{(k+1)} = W_i V_i^k + C_1 \times R_1 x (pbest - S_i^k) + C_2 \times R_2 x (gbest - S_i^k) \quad (19)$$

Where

- $V_i^k$  - current velocity of particle i at iteration k,
- $V_i^{(k+1)}$  - updated velocity of particle i,
- $W_i$  - different inertia weight of particle i ,
- $C_1, C_2$  - positive constants,
- $S_i^k$  - current position of particle i at inertia k,
- $R_1, R_2$  - random number between 0 and 1,

The new position can be modified using the present position and updated swarm position is

$$S_i^{(k+1)} = S_i^k + V_i^{(k+1)} \quad (20)$$

The parameter  $W_i$  is inertia weight that increases the overall performance of PSO. The larger value of  $W_i$  can favour higher ability for global search and lower value of  $W_i$  implies a higher ability for local search. To achieve a higher performance, the linearly decreased value of inertia is

proposed by You-Bo Wang .et.al [13] according the following formula.

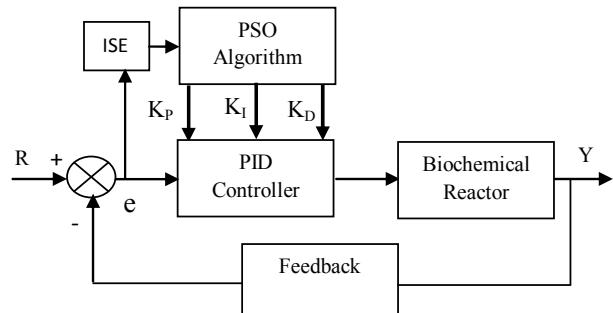


Fig. 3 Block diagram of PID control system combined with PSO

$$W = W_{\max} - Iter \times [(W_{\max} - W_{\min}) / (Iter_{\max})] \quad (21)$$

Where  $Iter_{\max}$  is the maximum of iteration in evolution process,  $W_{\max}$  is maximum value of inertia weight ,  $W_{\min}$  is the minimum value of inertia weight, and  $Iter$  is current value of iteration.

The simulation is carried out by using  $W_{\min}=0.3$  and  $W_{\max}=1.8$ . The iteration time required is more with the eqn.21 so an intermediate value of W is directly used.

The PSO algorithm is simulated with the following values. The positive constants  $C_1$  and  $C_2$  are set to 2 and 1.5 respectively. The inertia weight W is set as 0.7, size of the swarm is 25, and the dimension of the problem is 3 (ie.  $K_p$ ,  $K_i$  &  $K_d$ ). PSO is a search algorithm which finds the optimum controller values based on the Integral Square Error (ISE).

### C. Internal Model Controller

In this paper, the performance of the proposed PSO method is compared with an IMC and it has been shown to be a powerful method for control system synthesis. Recently, a modified IMC structure was proposed for unstable process with time delay [16]. In this method, the control structure can be tuned easily and the parameters to be tuned are  $K_0$ ,  $K_1$ ,  $K_2$  and  $\lambda$  (tuning parameter).

Tuning formulae are;

$$K_0 = 2 / K \quad (22)$$

$$K_1 = (\tau s + 1) / [K (\lambda s + 1)] \quad (23)$$

$$K_2 = (1/K) \times [(0.49 \tau / d) + 0.694] \quad (24)$$

## V. SIMULATION RESULTS AND DISCUSSIONS

### A. Case study:

Let us consider the unstable FOPDT process widely studied in the literature [1,8]

$$G(s) = \frac{e^{-0.1s}}{(s-1)}, \text{ A step test is carried out with a P controller.}$$

The controller value ' $K_c$ ' is varied to get the desired ' $\zeta$ '. From Fig.4, it is observed that the smaller ' $K_c$ ' can increase the ' $\zeta$ ' and the response of the system is less oscillatory. This response cannot be used for parameter estimation. The small ' $K_c$ ' can also increase the 'offset' due to the P

controller. It is necessary to identify the required ' $\zeta$ ' to get minimum offset and a good estimated model.

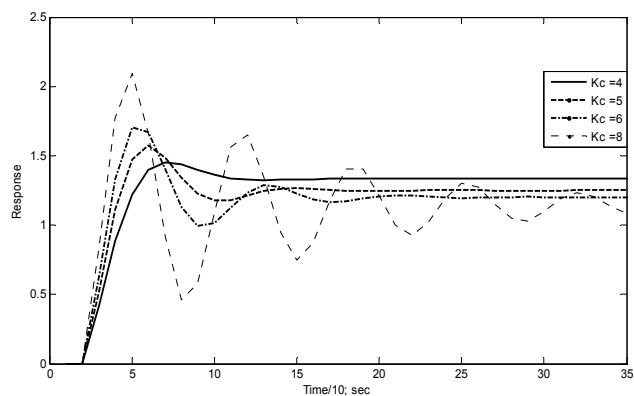


Fig.4. Step response of case study for different P controller values

Table.1. Step test data for the case study

<b>K<sub>c</sub></b>	<b>Y<sub>p</sub></b>	<b>Y<sub>v</sub></b>	<b>Y<sub>a</sub></b>	<b>Δt</b>	<b>V</b>	<b>P<sub>1</sub></b>	<b>P<sub>2</sub></b>	<b>Identified Model</b>
4	1.45	1.32	1.33	0.6	1.12	1.73	2.17	K = 0.9994, d = 0.304 and $\tau = 0.72$
5	1.57	1.18	1.25	0.4	1.12	1.99	2.28	K = 0.9993, d = 0.222 and $\tau = 0.58$
6	1.70	0.99	1.20	0.4	0.42	1.91	3.84	K = 0.9994, d = 0.304 and $\tau = 0.72$
8	2.25	0.06	1.13	0.3	0.94	2.83	3.05	K = 1, d = 0.107 and $\tau = 0.835$

The step test is performed with a step size of '1', sampling period of 0.1 sec and different 'K<sub>c</sub>' values. The offset can be minimized by increasing the controller gain. The response which gives more oscillation (a range of 3 to 5 peak and valley) can be considered to estimate the model parameters. The results are given in Table 1. From this it is inferred that a good matching is obtained between the process and model parameters for K<sub>c</sub> = 8. For unstable system with  $0 \leq D/\tau \leq 0.8$ , the recommended range for damping ratio is  $0.1 < \zeta < 0.45$ . This method can also be applied for the system with measurement noise.

#### B. Application to Biochemical reactor:

For the unstable biochemical reactor (Eqn.4), the step test with P controller is performed with a step size of 0.995103 (x<sub>1s</sub>) and sampling period of 0.1 sec. Table 2. shows the step test data and the identified model. A good model parameter is obtained when K<sub>c</sub> = - 0.85 with  $\zeta = 0.38$ . The model K = -6.016, d = 0.9925 and  $\tau = 5.723$  is then considered to tune the PSO based PID controller and the modified IMC controller.

Optimum PSO based PID parameters are K<sub>p</sub> = - 0.491, K<sub>i</sub> = - 0.05011 and K<sub>d</sub> = - 0.1201 and the modified IMC values are K<sub>0</sub> = - 0.33245, K<sub>1</sub> = (5.6231s+1) / (- 6.016s - 6.016) and K<sub>2</sub> = - 0.577.

Fig.5 and Fig.6 shows the regulatory and servo response for the model (Eqn.4) of the biochemical reactor. The PSO based PID gives a large overshoot for the step change due to proportional and derivative kick. So it can be replaced by an I-PD controller. The response of the I-PD is very smooth and it is similar to the modified IMC controller.

Table 3. shows the performance comparison between the controllers for a step input. PSO based I-PD controller gives the good performance like the IMC.

Table.2. Step test data for the Biochemical reactor

<b>K<sub>c</sub></b>	<b>Y<sub>p</sub></b>	<b>Y<sub>v</sub></b>	<b>Y<sub>a</sub></b>	<b>Δt</b>	<b>ζ</b>	<b>Identified Model</b>
-0.85	1.689	1.089	1.2431	2.8	0.38	K = -6.016, d = 0.9925 and $\tau = 5.723$
-1	1.836	0.875	1.198	2.6	0.23	K = -6.038, d = 1.194 and $\tau = 5.462$
-1.1	1.897	0.777	1.185	2.5	0.19	K = -6.053, d = 1.21 and $\tau = 5.46$

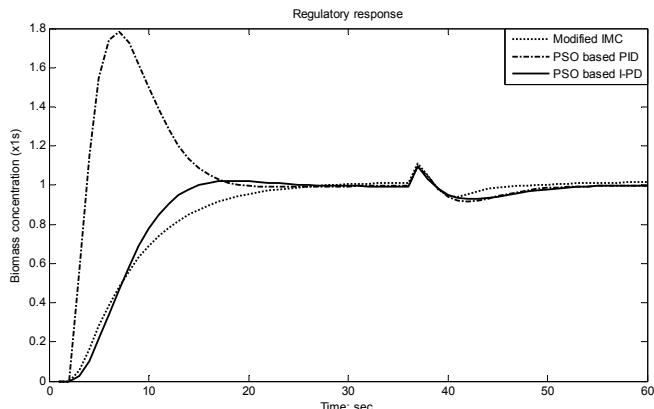


Fig.5. Regulatory response for the transfer function model

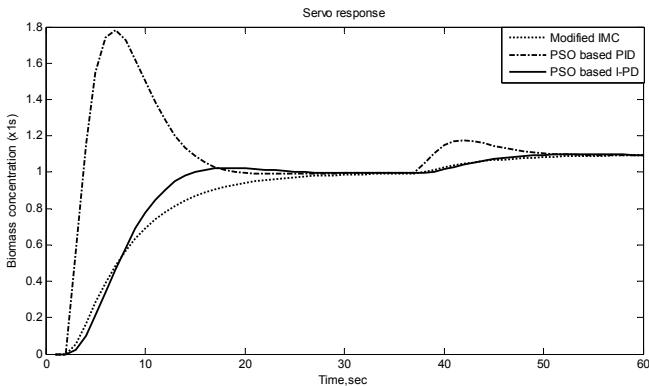


Fig.6. Servo response for the transfer function model

Table.3. Performance comparison

Performance Criteria	Modified IMC	PSO based PID	PSO based I - PD
Rise time (sec)	36.4	2.8	13.7
Overshoot	0	0.792	0.029
Settling time (sec)	39.7	24	28.5
ISE (Servo)	54.18	76.75	55.5
IAE (Servo)	54.51	65.91	55.18
ISE (Regulatory)	49.85	70.12	51.01
IAE (Regulatory)	52.44	62.74	52.98

Fig.7 shows the step response of the bioreactor model [2] with PID and PI controller (the controller gains for PI/PID are same as the PSO based PID). Due to the effect of the derivative controller, a kick effect is produced in the biomass concentration ( $x_{1s}$ ) and substrate concentration ( $x_{2s}$ ) with the PID controller. This kick effect can be reduced by using PI controller (with same controller gain). From Fig.7 it is noted that the PI controller shows a sluggish response than the PID controller.

Fig.8 shows the regulatory response of the system with PID controller with a change in the substrate feed of 0.4 at 35 sec and Fig.9 shows the servo response of the system with PID controller with a change in the biomass concentration of 0.1 at 35sec. In both the cases the proposed PSO based PID controller gives the satisfactory response.

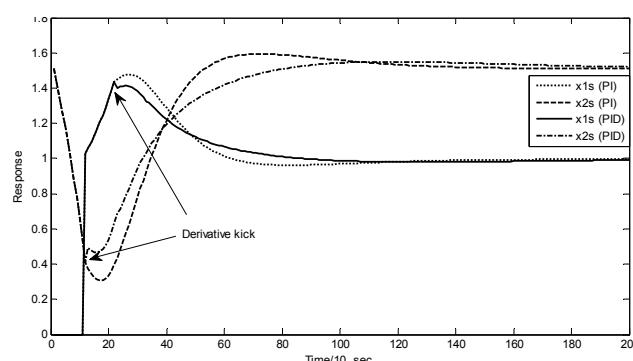


Fig.7. Step response of the bioreactor with PID and PI controller

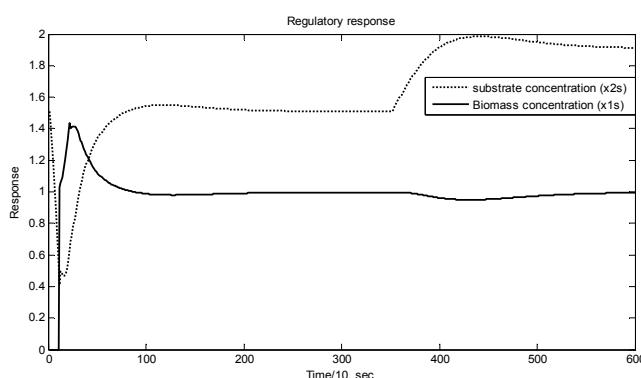


Fig.8. Regulatory response for the biochemical reactor

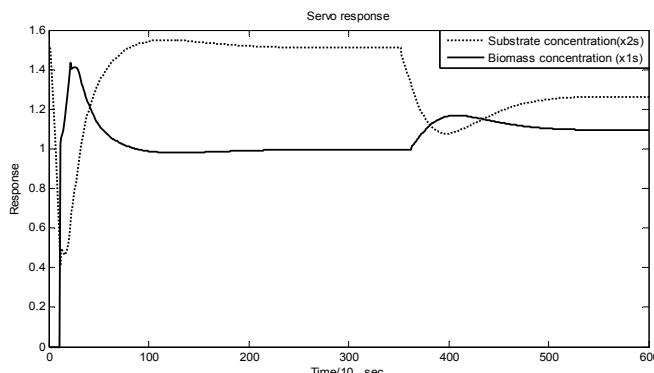


Fig.9. Servo response for the biochemical reactor

## VI. CONCLUSION

In this work, the system identification using a simple step response with a P controller is utilized for unstable biochemical reactor model. The proposed method is simple, easy to use and the model accuracy is the function of damping ratio (' $\zeta$ '). For unstable system with  $0 \leq D/\tau \leq 0.8$ , the recommended range for damping ratio is  $0.1 < \zeta < 0.45$ . A PSO based PID controller is also proposed for the biochemical reactor. The simulation result shows that the proposed method gives good result for the servo and regulatory responses with the transfer function model and the biochemical reactor model. The important contribution of this work is, the model accuracy of the identified system can be improved by selecting a suitable range of ' $\zeta$ '. The effect of proportional and derivative kick can be minimized using a modified PID structure with optimum controller tuning parameters.

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